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One Raised Product Prime Labeling of Some Snake Graphs

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Abstract- One raised product prime labeling of a graph is the labeling of the vertices with {1,2---,p} and the edges with product of the labels of the incident vertices plus 1. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits one raised product prime labeling. Here we investigate some snake graphs for one raised product prime labeling.

Index Terms- Graph labeling; product; prime labeling; prime graphs; snake graphs**.**

1. INTRODUCTION

All graphs in this paper are simple,finite and undirected. The symbol V(G) and E(G) denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4] . Some basic concepts are taken from Frank Harary [1]. In this paper we investigated one raised product prime labeling of some snake graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2. MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection

f : V(G) → {1,2,---, p} by $f(v_i) = i$, for every i from 1 to p and define a 1-1 mapping f_{orpp}^* : E(G) → set of natural numbers N by $f_{orpp}^*(uv) = f(u)f(v) + 1$ .The induced function f_{orpp}^* is said to be one raised product prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

Definition 2.2 A graph which admits one raised product prime labeling is called one raised product prime graph.

Theorem 2.1 Triangular snake T_n admits one raised product prime labeling.

Proof: Let G = T_n and let $v_1, v_2, ---, v_{2n-1}$ are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 3n-3.

Define a function f : V → {1,2,---,2n-1} by

$$f(v_i) = i , i = 1,2,---,2n-1$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{orpp}^* is defined as follows

$f_{orpp}^*(v_i\, v_{i+1})$ = i^2+i+1, i = 1,2,---,2n-2.

$f_{orpp}^*(v_{2i-1}\, v_{2i+1})$ = $4i^2$, i = 1,2,---,n-1.

Clearly f_{orpp}^* is an injection.

gcin of (v_1) = gcd of $\{f_{orpp}^*(v_1\, v_2), f_{orpp}^*(v_1\, v_3)\}$
= gcd of {3, 4}= 1.

gcin of (v_{i+1}) = gcd of $\{f_{orpp}^*(v_i\, v_{i+1}), f_{orpp}^*(v_{i+1}\, v_{i+2})\}$
= gcd of $\{i^2+i+1, i^2+3i+3\}$
= gcd of {2i+2,i(i+1)+1}
= gcd of {i+1,i(i+1)+1}
= 1, i = 1,2,---,2n-3.

gcin of (v_{2n-1}) = gcd of $\{f_{orpp}^*(v_{2n-2}\, v_{2n-1}), f_{orpp}^*(v_{2n-3}\, v_{2n-1})\}$
= gcd of $\{(2n-2)(2n-1)+1, (2n-2)^2\}$
= gcd of {2n-2, (2n-2)(2n-1)+1}
= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence T_n, admits one raised product prime labeling.

Example 2.1 G = T_4.

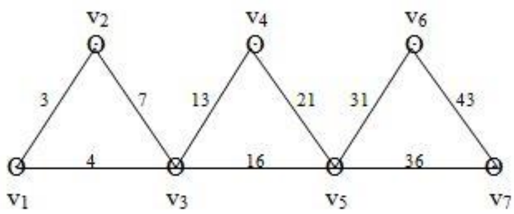

fig – 2.1

Theorem 2.2 Quadrilateral snake Q_n admits one raised product prime labeling.

Proof: Let G = Q_n and let $v_1, v_2, ---, v_{3n-2}$ are the vertices of G.

Here |V(G)| = 3n-2 and |E(G)| = 4n-4.

Define a function $f : V \rightarrow \{1,2,---,3n-2\}$ by
$f(v_i) = i$, $i = 1,2,---,3n-2$
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_i\ v_{i+1})$ = i^2+i+1, $i = 1,2,---,3n-3$.
$f^*_{orpp}(v_{3i-2}\ v_{3i+1})$ = $9i^2-3i-1$, $i = 1,2,---,n-1$.

Clearly f^*_{orpp} is an injection.

gcin of (v_1) = gcd of $\{f^*_{orpp}(v_1\ v_2), f^*_{orpp}(v_1\ v_4)\}$
= gcd of {3, 5}= 1.

gcin of (v_{i+1}) = gcd of $\{f^*_{orpp}(v_i\ v_{i+1}), f^*_{orpp}(v_{i+1}\ v_{i+2})\}$
= 1, $i = 1,2,---,3n-4$.

gcin of (v_{3n-2}) = gcd of $\{f^*_{orpp}(v_{3n-2}\ v_{3n-5}), f^*_{orpp}(v_{3n-2}\ v_{3n-3})\}$
= gcd of { (3n-2)(3n-5)+1, (3n-2)(3n-3)+1}
= gcd of {2(3n-2), (3n-2)(3n-5)+1}
= gcd of {(3n-2), (3n-2)(3n-5)+1}
=1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence Q_n, admits one raised product prime labeling.

Example 2.2 G = Q_4.

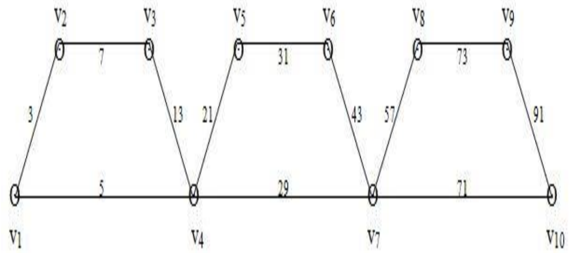

fig – 2.2

Theorem 2.3 Alternate triangular snake $A(T_n)$ admits one raised product prime labeling, if n is odd and triangle starts from the first vertex.

Proof: Let G = $A(T_n)$ and let $v_1,v_2,---,v_{\frac{3n-1}{2}}$ are the vertices of G.

Here $|V(G)| = \frac{3n-1}{2}$ and $|E(G)| = 2n-2$.

Define a function $f : V \rightarrow \{1,2,---, \frac{3n-1}{2}\}$ by
$f(v_i) = i$, $i = 1,2,---, \frac{3n-1}{2}$
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_i\ v_{i+1})$ = i^2+i+1, $i = 1,2,---, \frac{3n-3}{2}$.
$f^*_{orpp}(v_{3i-2}\ v_{3i})$ = $9i^2-6i+1$, $i = 1,2,---, \frac{n-1}{2}$.

Clearly f^*_{orpp} is an injection.

gcin of (v_1) = gcd of $\{f^*_{orpp}(v_1\ v_2), f^*_{orpp}(v_1\ v_3)\}$
= gcd of {3, 4}
= 1.

gcin of (v_{i+1}) = gcd of $\{f^*_{orpp}(v_i\ v_{i+1}), f^*_{orpp}(v_{i+1}\ v_{i+2})\}$
= 1, $i = 1,2,---, \frac{3n-5}{2}$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $A(T_n)$, admits one raised product prime labeling.

Example 2.3 G = $A(T_5)$.

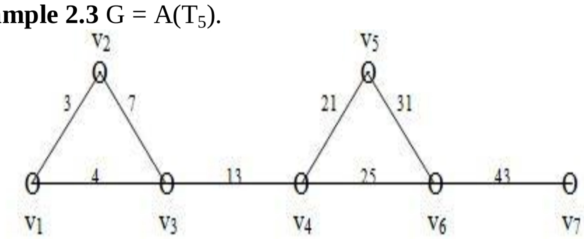

fig – 2.3

Theorem 2.4 Alternate triangular snake $A(T_n)$ admits one raised product prime labeling, if n is even and triangle starts from the second vertex.

Proof: Let G = $A(T_n)$ and let $v_1,v_2,---,v_{\frac{3n-2}{2}}$ are the vertices of G.

Here $|V(G)| = \frac{3n-2}{2}$ and $|E(G)| = 2n-3$.

Define a function $f : V \rightarrow \{1,2,---, \frac{3n-2}{2}\}$ by
$f(v_i) = i$, $i = 1,2,---, \frac{3n-2}{2}$
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_i\ v_{i+1})$ = i^2+i+1, $i = 1,2,---, \frac{3n-4}{2}$.
$f^*_{orpp}(v_{3i-1}\ v_{3i+1})$ = $9i^2$, $i = 1,2,---, \frac{n-2}{2}$.

Clearly f^*_{orpp} is an injection.

gcin of (v_{i+1}) = gcd of $\{f^*_{orpp}(v_i\ v_{i+1}), f^*_{orpp}(v_{i+1}\ v_{i+2})\}$
= 1, $i = 1,2,---, \frac{3n-6}{2}$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $A(T_n)$, admits one raised product prime labeling.

Example 2.4 G = $A(T_5)$.

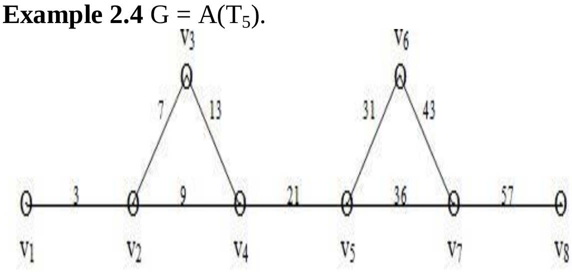

fig – 2.4

Theorem 2.5 Alternate quadrilateral snake graph $A(Q_n)$ admits one raised product prime labeling, if n is odd and quadrilateral starts from the first vertex.

Proof: Let G = $A(Q_n)$ and let $v_1,v_2,---,v_{2n-1}$ are the vertices of G.

Here $|V(G)| = 2n-1$ and $|E(G)| = \frac{5n-5}{2}$.

Define a function $f : V \rightarrow \{1,2,---,2n-1\}$ by
$f(v_i) = i$, $i = 1,2,---,2n-1$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_{4i-3}\ v_{4i-2}) = 16i^2-20i+7$, $i = 1,2,---,\frac{n-1}{2}$.

$f^*_{orpp}(v_{4i-3}\ v_{4i}) = 16i^2-12i+1$, $i = 1,2,---,\frac{n-1}{2}$.

$f^*_{orpp}(v_{4i-2}\ v_{4i-1}) = 16i^2-12i+3$, $i = 1,2,---,\frac{n-1}{2}$.

$f^*_{orpp}(v_{4i-1}\ v_{4i}) = 16i^2-4i+1$, $i = 1,2,---,\frac{n-1}{2}$.

$f^*_{orpp}(v_{4i}\ v_{4i+1}) = 16i^2+4i+1$, $i = 1,2,---,\frac{n-1}{2}$.

Clearly f^*_{orpp} is an injection.

gcin of (v_1) = gcd of $\{f^*_{orpp}(v_1\ v_2), f^*_{orpp}(v_1\ v_4)\}$
= gcd of {3, 5}= 1.

gcin of (v_{i+1}) = gcd of $\{f^*_{orpp}(v_i\ v_{i+1}), f^*_{orpp}(v_{i+1}\ v_{i+2})\}$
= 1, $i = 1,2,---,2n-3$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $A(Q_n)$, admits one raised product prime labeling.

Example 2.5 G = $A(Q_5)$.

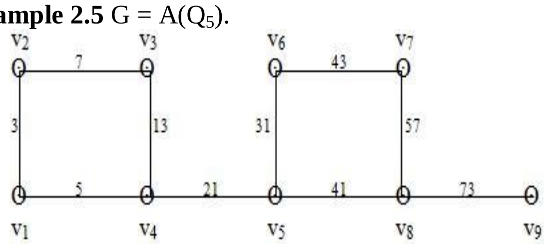

fig -2.5

Theorem 2.6 Alternate quadrilateral snake graph $A(Q_n)$ admits one raised product prime labeling, if n is even and quadrilateral starts from the second vertex.

Proof: Let G = $A(Q_n)$ and let $v_1,v_2,---,v_{2n-2}$ are the vertices of G.

Here $|V(G)| = 2n-2$ and $|E(G)| = \frac{5n-8}{2}$.

Define a function $f : V \rightarrow \{1,2,---,2n-2\}$ by
$f(v_i) = i$, $i = 1,2,---,2n-2$.

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_{4i-3}\ v_{4i-2}) = 16i^2-20i+7$, $i = 1,2,---,\frac{n}{2}$.

$f^*_{orpp}(v_{4i-2}\ v_{4i+1}) = 16i^2-4i-1$, $i = 1,2,---,\frac{n-2}{2}$.

$f^*_{orpp}(v_{4i-2}\ v_{4i-1}) = 16i^2-12i+3$, $i = 1,2,---,\frac{n-2}{2}$.

$f^*_{orpp}(v_{4i-1}\ v_{4i}) = 16i^2-4i+1$, $i = 1,2,---,\frac{n-2}{2}$.

$f^*_{orpp}(v_{4i}\ v_{4i+1}) = 16i^2+4i+1$, $i = 1,2,---,\frac{n-2}{2}$.

Clearly f^*_{orpp} is an injection.

gcin of (v_{i+1}) = gcd of $\{f^*_{orpp}(v_i\ v_{i+1}), f^*_{orpp}(v_{i+1}\ v_{i+2})\}$
= 1, $i = 1,2,---,2n-4$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $A(Q_n)$, admits one raised product prime labeling.

Example 2.6 G = $A(Q_6)$.

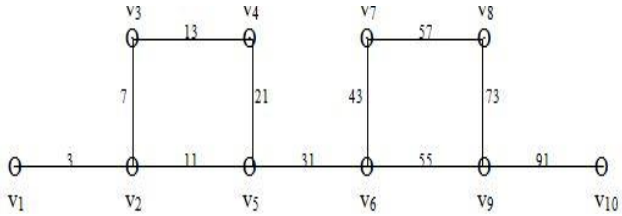

fig – 2.6

Theorem 2.7 Alternate double triangular snake graph $A(DT_n)$ admits one raised product prime labeling, if n is even and double triangle starts from the second vertex.

Proof: Let G = $A(DT_n)$ and let $v_1,v_2,---,v_{2n-2}$ are the vertices of G.

Here $|V(G)| = 2n-2$ and $|E(G)| = 3n-5$.

Define a function $f : V \rightarrow \{1,2,---,2n-2\}$ by
$f(v_i) = i$, $i = 1,2,---,2n-2$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_{4i-2}\ v_{4i-1}) = 16i^2-12i+3$, $i = 1,2,---,\frac{n-2}{2}$.

$f^*_{orpp}(v_{4i-1}\ v_{4i}) = 16i^2-4i+1$, $i = 1,2,---,\frac{n-2}{2}$.

$f^*_{orpp}(v_{4i}\ v_{4i+1}) = 16i^2+4i+1$, $i = 1,2,---,\frac{n-2}{2}$.

$f^*_{orpp}(v_{4i-2}\ v_{4i+1}) = 16i^2-4i-1$, $i = 1,2,---,\frac{n-2}{2}$.

$f^*_{orpp}(v_{2i}\ v_{2i+2}) = 4i^2+4i+1$, $i = 1,2,---,n-2$.

$f^*_{orpp}(v_1\ v_2) = 3$.

Clearly f^*_{orpp} is an injection.

gcin of (v_{4i-2}) = gcd of $\{f^*_{orpp}(v_{4i-2}\ v_{4i-1}), f^*_{orpp}(v_{4i-2}\ v_{4i+1})\}$
= gcd of {(4i-2)(4i-1)+1, (4i-2)(4i+1)+1}
= gcd of {2(4i-2), (4i-2)(4i-1)+1}
= gcd of {(4i-2), (4i-2)(4i-1)+1}
= 1, $i = 1,2,---, \frac{n-2}{2}$.

gcin of (v_{4i-1}) = gcd of $\{f^*_{orpp}(v_{4i-2}\ v_{4i-1}), f^*_{orpp}(v_{4i-1}\ v_{4i})\}$

= gcd of {(4i-2)(4i-1)+1, (4i-1)(4i)+1}
= gcd of {2(4i-1), (4i-2)(4i-1)+1}
= gcd of {(4i-1), (4i-2)(4i-1)+1}
= 1, i = 1,2,---, $\frac{n-2}{2}$.

gcin of (v_{4i}) = gcd of {$f^*_{orpp}(v_{4i-1}\ v_{4i})$, $f^*_{orpp}(v_{4i}\ v_{4i+1})$}
= gcd of {(4i-1)(4i)+1, (4i)(4i+1)+1}
= gcd of {8i, (4i)(4i-1)+1}
= gcd of {(4i), (4i)(4i-1)+1}
= 1, i = 1,2,---, $\frac{n-2}{2}$.

gcin of (v_{4i+1}) = gcd of {$f^*_{orpp}(v_{4i-2}\ v_{4i+1})$, $f^*_{orpp}(v_{4i}\ v_{4i+1})$}
= gcd of { (4i-2)(4i+1)+1, (4i+1)(4i)+1}
= gcd of {2(4i+1), (4i-2)(4i+1)+1}
= gcd of {(4i+1), (4i+1)(4i-2)+1}
= 1, i = 1,2,---, $\frac{n-2}{2}$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence A(DT_n), admits one raised product prime labeling.

Example 2.7 G = A(DT_6).

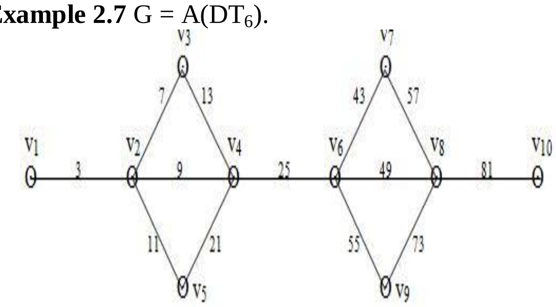

fig – 2.7

Theorem 2.8 Alternate double triangular snake graph A(DT_n) admits one raised product prime labeling, if n is even and double triangle starts from the first vertex.

Proof: Let G = A(DT_n) and let $v_1,v_2,---,v_{2n}$ are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 3n-1.
Define a function f : V → {1,2,---,2n} by

$f(v_i)$ = i , i = 1,2,---,2n.

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_{4i-3}\ v_{4i-2})$ = $16i^2$-20i+7, i = 1,2,---,$\frac{n}{2}$.

$f^*_{orpp}(v_{4i-3}\ v_{4i-1})$ = $16i^2$-16i+4, i = 1,2,---,$\frac{n}{2}$.

$f^*_{orpp}(v_{4i-3}\ v_{4i})$ = $16i^2$-12i+1, i = 1,2,---,$\frac{n}{2}$.

$f^*_{orpp}(v_{4i-2}\ v_{4i-1})$ = $16i^2$-12i+3, i = 1,2,---,$\frac{n}{2}$.

$f^*_{orpp}(v_{4i-1}\ v_{4i})$ = $16i^2$-4i+1, i = 1,2,---,$\frac{n}{2}$.

$f^*_{orpp}(v_{4i-1}\ v_{4i+1})$ = $16i^2$, i = 1,2,---,$\frac{n-2}{2}$.

Clearly f^*_{orpp} is an injection.

gcin of (v_{4i-2}) = gcd of {$f^*_{orpp}(v_{4i-3}\ v_{4i-2})$, $f^*_{orpp}(v_{4i-2}\ v_{4i-1})$}
= gcd of {$16i^2$-20i+7, $16i^2$-12i+3}
= gcd of {8i-4, $16i^2$-20i+7}
= gcd of {4i-2), (4i-3)(4i-2)+1}
= 1, i = 1,2,---, $\frac{n}{2}$.

gcin of (v_{4i-1}) = gcd of {$f^*_{orpp}(v_{4i-3}\ v_{4i})$, $f^*_{orpp}(v_{4i-1}\ v_{4i})$}
= gcd of {$16i^2$-12i+3, $16i^2$-4i+1}
= gcd of {8i-2, $16i^2$-12i+3}
= gcd of {4i-1), (4i-1)(4i-2)+1}
= 1, i = 1,2,---, $\frac{n}{2}$.

gcin of (v_{4i}) = gcd of {$f^*_{orpp}(v_{4i-3}\ v_{4i})$, $f^*_{orpp}(v_{4i-1}\ v_{4i})$}
= gcd of {$16i^2$-12i+1, $16i^2$-4i+1}
= gcd of {8i, $16i^2$-12i+1}
= gcd of {4i, (4i-3)(4i)+1}
= 1, i = 1,2,---, $\frac{n}{2}$.

gcin of (v_{4i-3}) = gcd of {$f^*_{orpp}(v_{4i-3}\ v_{4i-2})$, $f^*_{orpp}(v_{4i-3}\ v_{4i-1})$}
= gcd of {$16i^2$-20i+7, $16i^2$-16i+4}
= gcd of {4i-3, $16i^2$-20i+7}
= gcd of {4i-3), (4i-3)(4i-2)+1}
= 1, i = 1,2,---, $\frac{n}{2}$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence A(DT_n), admits one raised product prime labeling.

Example 2.8 G = A(DT_4).

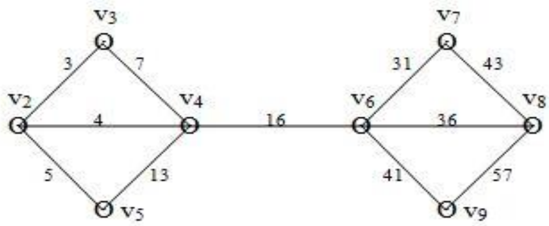

fig – 2.8

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